Effect of arbitrary magnetic Reynolds number on MHD flows in convergent-divergent channels

O.D. Makinde
Applied Mathematics Department, University of Limpopo, Limpopo, South Africa

Received 15 December 2006
Revised 10 July 2007
Accepted 10 July 2007

Abstract

Purpose – The objective of the present study is to investigate the effect of arbitrary magnetic Reynolds number on steady flow of an incompressible conducting viscous liquid in convergent-divergent channels under the influence of an externally applied homogeneous magnetic field.

Design/methodology/approach – The solution of the non-linear 2D Navier-Stokes equations modeling the flow field is obtained using a perturbation technique coupled with a special type of Hermite-Padé approximation method implemented numerically on MAPLE and a bifurcation study is performed.

Findings – The results show that increasing values of magnetic Reynolds number causes a general decrease in the fluid velocity around the central region of the channel. The flow reversal control is also observed by increasing magnetic field intensity. The bifurcation study reveals the solution branches and turning points.

Practical implications – The reported results are very useful in the field of engineering flow control and industrial metal casting for the control of molten metal flows.

Originality/value – Effect of arbitrary magnetic Reynolds on the overall flow structure in converging-diverging channels are presented and studied using a newly developed numerical approach.

Keywords Flow, Liquid flow, Number theory, Viscosity, Inertia, Magnetism

Paper type Research paper

1. Introduction

In modern times, the theory of flow through convergent-divergent channels has many applications in aerospace, chemical, civil, environmental, mechanical and bio-mechanical engineering as well as in understanding rivers and canals. The mathematical investigations of this type of problem were pioneered by Jeffery (1915) and Hamel (1916), and have been extensively studied by several authors and discussed in many textbooks (Fraenkel, 1962; Batchelor, 1967; Sobey and Drazin, 1986; Banks *et al.*, 1988; Hamadiche *et al.*, 1994; Makinde, 1997, 1999; Khan *et al.*, 2003; Makinde and Mhone, 2007). Jeffery-Hamel flows are interesting models of the phenomenon of separation of boundary layers in divergent channels. These flows have revealed a multiplicity of solutions, richer perhaps than other similarity solutions of the Navier-Stokes equations, no doubt because of the dependence on two non-dimensional parameters, i.e. the flow Reynolds number and channel angular widths.

The author would like to thank anonymous referees for their useful suggestions and the National Research Foundation of South Africa Thuthuka programme for financial support.

International Journal of Numerical Methods for Heat & Fluid Flow
Vol. 18 No. 6, 2008
pp. 697-707

0961-5539
DOI 10.1108/09615530810885524

Meanwhile, the study of electrically conducting viscous fluid flowing through convergent-divergent channels under the influence of an external magnetic field is not only fascinating theoretically but also finds applications in mathematical modelling of several industrial and biological systems. A possible practical application of the theory we envisage is in the field of industrial metal casting, the control of molten metal flows. Another area in which the theoretical study may be of interest is in the motion of liquid metals or alloys in the cooling systems of advanced nuclear reactors. Clearly, the motion in the region with intersecting walls may represent a local transition between two parallel channels with different cross-sections, a widening or a contraction of the flow. A survey of magneto-hydrodynamics (MHD) studies in the mentioned technological field can be found in Moreau (1990). The problem is basically an extension of classical Jeffery-Hamel flows of ordinary fluid mechanics to MHD. In the MHD solution, an external magnetic field acts as a control parameter for both convergent and divergent channel flows. Here, beside the flow Reynolds number (R) and the angle of the walls, two other non-dimensional parameters determine the solutions, the magnetic Reynolds number (Rm), and the Hartmann number (H). Hence, a much larger variety of solutions than in the classical problem are expected.

This paper extends the recent bifurcation study of Makinde and Mhone (2006) on MHD Jeffery-Hamel flows to include arbitrary magnetic Reynolds number. The main results is that for H larger than 2, the flow reversal near the walls, which is a typical feature of the classical Jeferry-Hamel flows without magnetic field and moderately large Reynolds number, tends to disappear. Moderate values of $H = 4$ are sufficient to suppress flow reversal up to $R = 7$ for channel semi-angle as large as 45°. We also observed that turning points whose magnitude depends on various flow parameters exist in the flow field. In Sections 2 and 3, we establish the mathematical formulation for the problem. Computer extension of the resulting perturbation series solution and the bifurcation study are conducted using a special type of Hermite-Pade approximation technique in Section 4. In Section 5, we discuss the entire findings.

2. Mathematical formulation

Consider the steady 2D flow of an incompressible conducting viscous fluid from a source or sink at the intersection between two rigid plane walls.

An external current flowing along the line where the walls meet is used to generate a magnetic field exerting a body force that controls the flow. In the laboratory frame of reference, it is assumed that the induced electric field is negligible, hence, the Ohm's law reduces to $\tilde{J} = \sigma(\tilde{\mathbf{q}} \times \tilde{B})$, where $\tilde{J}$ is the current density, σ the conductivity of the fluid, $\tilde{B} = (B_r, B_\theta)$ the magnetic flux density, and $\tilde{\mathbf{q}} = (u, v)$ is the velocity vector with components in the radial and tangential direction, respectively. Let (r, θ) be polar coordinate with $r = 0$ as the sink or source, α the semi-angle and the domain of the flow is represented by $-|\alpha| < \theta < |\alpha|$. Then, the MHD governing equations in terms of the vorticity (ω) and stream-function (Ψ) formulation with Ampere's law are given as (Figure 1):

$$\frac{1}{r}\frac{\partial(\Psi, \omega)}{\partial(\theta, r)} = \upsilon\nabla^2\omega + \nabla \times \left(\frac{1}{\rho}(\tilde{J} \times \tilde{B})\right), \quad \omega = -\nabla^2\Psi, \tag{1}$$

$$\nabla \cdot \tilde{B} = 0, \quad \nabla \times \tilde{B} = \mu_e \tilde{J}, \tag{2}$$

with:

$$\Psi = \pm \frac{Q}{2}, \quad \frac{\partial \Psi}{\partial \theta} = 0, \quad \text{at } \theta = \pm\alpha, \tag{3}$$

where:

$$Q = \int_{-\alpha}^{\alpha} ur \mathrm{d}\theta, \tag{4}$$

is the volumetric flow rate, μ_e the magnetic permeability, ρ the fluid density, $\nabla^2 = \partial^2/\partial r^2 + \partial/r\partial r + \partial^2/r^2\partial\theta^2$ and υ is the kinematic viscosity coefficient. For Jeffery-Hamel flow of conducting fluid, we assume a purely symmetric radial flow (Banks *et al.*, 1988), so that the tangential velocity $v = 0$ and as a consequence of the mass conservation and the non-existence of magnetic monopoles (equation (2)), we have the stream-function given by $\Psi = QG(\theta)/2$, $B_r = 2K/r$ and $B_\theta = \mathrm{d}F/r\mathrm{d}\theta$, where $F(\theta)$ is the magnetic flux and K is a constant representing electric current flowing in the z direction. If we require $Q \geq 0$ then for $\alpha < 0$ the flow is converging to a sink at $r = 0$. The dimensionless form of equations (1)-(3) now become:

$$\frac{\mathrm{d}^4G}{\mathrm{d}y^4} + 2R\alpha\frac{\mathrm{d}G}{\mathrm{d}y}\frac{\mathrm{d}^2G}{\mathrm{d}y^2} + (4-H)\alpha^2\frac{\mathrm{d}^2G}{\mathrm{d}y^2} + 2H^2Rm\alpha^3G\frac{\mathrm{d}G}{\mathrm{d}y} = 0, \tag{5}$$

with:

$$G = \pm 1, \quad \frac{\mathrm{d}G}{\mathrm{d}y} = 0, \quad \text{at } y = \pm 1, \tag{6}$$

where $y = \theta/\alpha$ and $H = 2K\sqrt{\sigma/\rho\upsilon}$, $R = Q/2\upsilon$, $Rm = \mu_e\sigma Q$ are the Hartmann number, the flow Reynolds number and the magnetic Reynolds number, respectively.

3. Perturbation method

The problem posed by equations (5) and (6) is non-linear, for small channel angular width, we shall seek asymptotic expansion of the form:

$$G(y) = \sum_{i=0}^{\infty} \alpha^i G_i. \tag{7}$$

Substituting the above expressions (7) into equations (5) and (6) and collecting the coefficients of like powers of α we obtained and solved the equations governing G. Since it seems cumbersome to obtain many terms of the solution series manually, we

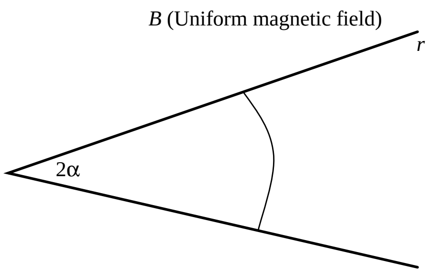


Figure 1. Schematic description of the channel

have written a MAPLE program that calculates successively the coefficients of the solution series. Some of the solution for stream-function and radial velocity obtained are given as follows:

$$
\begin{aligned}
&G(y,\alpha,R,Rm,H)\\
&=\frac{3}{2}y-\frac{1}{2}y^3-\frac{3}{280}Ry(y^2-5)(y^2-1)^2\alpha-\frac{1}{431,200}y(y-1)^2(y+1)^2\\
&\times(98R^2y^6-959R^2y^4+2,472y^2R^2-43,120+10,780H^2-2,875R^2)\alpha^2\\
&+\Bigg(-\frac{33}{5,600}yRH^2-\frac{41}{2,800}H^2Ry^5+\frac{1}{140}RmH^2y^7+\frac{1}{175}H^2Ry^7\\
&+\frac{1}{18}y^3RmH^2-\frac{3}{80}RmH^2y^5+\frac{11}{700}y^3RH^2-\frac{1}{1,120}H^2Ry^9\\
&-\frac{83}{3,360}yRmH^2-\frac{1}{2,016}RmH^2y^9+\frac{1}{280}Ry^9-\frac{4}{175}Ry^7+\frac{41}{700}Ry^5\\
&-\frac{1}{208,000}R^3y^{15}+\frac{127}{1,601,600}R^3y^{13}-\frac{603}{1,232,000}R^3y^{11}+\frac{31}{19,600}R^3y^9\\
&-\frac{184,599}{60,368,000}R^3y^7+\frac{16,493}{4,312,000}R^3y^5-\frac{11}{175}y^3R-\frac{439,093}{156,956,800}y^3R^3\\
&+\frac{33}{1,400}yR+\frac{33,897}{39,239,200}yR^3\Bigg)\alpha^3+O(\alpha^4)
\end{aligned}
\tag{8}
$$

$$
\begin{aligned}
&u(y,\alpha,R,Rm,H)\\
&=-\frac{3}{2}(y-1)(y+1)-\frac{3}{280}R(y-1)(y+1)(7y^4-28y^2+5)\alpha\\
&-\frac{1}{431,200}(y-1)(y+1)(1,078y^8R^2-9,317R^2y^6+22,099R^2y^4\\
&-21,791y^2R^2+53,900y^2H^2-215,600y^2+43,120+2,875R^2\\
&-10,780H^2)\alpha^2-\frac{1}{2,354,352,000}(y-1)(y+1)(169,785R^3y^{12}\\
&-2,257,185R^3y^{10}+10,418,478R^3y^8-23,095,002R^3y^6\\
&+10,510,500RmH^2y^6-75,675,600Ry^6+18,918,900H^2Ry^6\\
&+27,300,525R^3y^4-75,255,180H^2Ry^4-107,207,100RmH^2y^4\\
&+301,020,720Ry^4-17,725,365y^2R^3+97,117,020y^2RH^2\\
&+334,233,900y^2RmH^2-388,468,080y^2R+2,033,820R^3\\
&-58,158,100RmH^2+55,495,440R-13,873,860RH^2)\alpha^3+O(\alpha^4)
\end{aligned}
\tag{9}
$$

Using a computer symbolic algebra package (MAPLE), the first few terms of the above solution series in equations (8) and (9) as well as the series for the centreline velocity profile $u(0, \alpha, R, Rm, H)$ are obtained. We are aware that these power series solutions are valid for very small parameter values. However, using Hermite-Padé approximation technique, we have extended the usability of the solution series beyond small parameter values as illustrated in the following section.

4. Hermite-Padé approximation method

The main tool of this paper is a simple technique of series summation based on the generalization of Padé approximation technique and may be described as follows. Let us suppose that the partial sum:

$$U_{N-1}(\lambda) = \sum_{i=0}^{N-1} a_i \lambda^i = U(\lambda) + O(\lambda^N) \quad \text{as} \ \lambda \to 0, \tag{10}$$

is given (λ could be α or R). The accuracy attainable by equation (10) is often inadequate, and, indeed, the series may diverge (Baker and Graves-Morris, 1996). However, we are concerned with the bifurcation study by analytic continuation as well as the dominant behaviour of the solution by using partial sum in equation (10). We expect that the accuracy of the critical parameters will ensure the accuracy of the solution. It is well known that the dominant behaviour of a solution of a differential equation can often be written as Guttamann (1989):

$$U(\lambda) \approx \begin{cases} P(\lambda_c - \lambda)^m & \text{for} \ m \neq 0, 1, 2, \ldots \\ P(\lambda_c - \lambda)^m \ln|\lambda_c - \lambda| & \text{for} \ m = 0, 1, 2, \ldots \end{cases} \quad \text{as} \ \lambda \to \lambda_c, \tag{11}$$

where P is some constant and λ_c is the critical point with the exponent m. However, we shall make the simplest hypothesis with respect to non-linear problems by assuming that $U(\lambda)$ is the local representation of an algebraic function of λ. Therefore, we seek an expression of the form:

$$F_d(\lambda, U_{N-1}) = A_{0N}(\lambda) + A_{1N}^d(\lambda)U^{(1)} + A_{2N}^d(\lambda)U^{(2)} + A_{3N}^d(\lambda)U^{(3)}, \tag{12}$$

such that:

$$A_{0N}(\lambda) = 1, \quad A_{iN}^{\mathrm{d}}(\lambda) = \sum_{j=1}^{d+i} b_{ij}\lambda^{j-1} \tag{13}$$

and:

$$F_d(\lambda, U) = O(\lambda^{N+1}) \quad \text{as} \ \lambda \to 0, \tag{14}$$

where $d \geq 1, i = 1, 2, 3$. The asymptotic behaviour of the equation (10) with respect to the approximant in equation (12) has been fully discussed in the earlier papers by Sergeyev and Goodson (1998) and Tourigny and Drazin (2000). The condition (13) normalizes the F_d and ensures that the order of series A_{iN}^d increases as i and d increase in value. There are thus $3(2 + d)$ undetermined coefficients b_{ij} in the expression (13). The requirement equation (14) reduces the problem to a system of N linear equations for the unknown coefficients of F_d. The entries of the underlying matrix depend only on the N given coefficients a_i. Henceforth, we shall take:

$$N = 3(2 + d), \tag{15}$$

so that the number of equations equals the number of unknowns. Equation (12) is a new special type of Hermite-Padé approximants. Both the algebraic and differential approximants form of equation (12) are considered. For instance, we let:

$$U^{(1)} = U, \quad U^{(2)} = U^2, \quad U^{(3)} = U^3, \tag{16}$$

and obtain a cubic Padé approximant. This enables us to obtain solution branches of the underlying problem in addition to the one represented by the original series. In the same manner, we let:

$$U^{(1)} = U, \quad U^{(2)} = DU, \quad U^{(3)} = D^2U \tag{17}$$

in equation (12), where D is the differential operator given by $D = d/d\lambda$. This leads to a second order differential approximants. It is an extension of the integral approximants idea by Hunter and Baker (1979) and enables us to obtain the dominant singularity in the flow field, i.e. by equating the coefficient $A_{3N}(\lambda)$ in the equation (12) to zero. Meanwhile, it is very important to note that the rationale for chosen the degrees of A_{iN} in equation (13) in this particular application is based on the simple technique of singularity determination in second order linear ordinary differential equations with polynomial coefficients as well as the possibility of multiple solution branches for the non-linear problem (Vainberg and Trenogin, 1974). In practice, one usually finds that the dominant singularities are located at zeroes of the leading polynomial $A_{3N}^{(d)}$ coefficients of the second order linear ordinary differential equation. Hence, some of the zeroes of $A_{3N}^{(d)}$ may provide approximations of the singularities of the series U and we expect that the accuracy of the singularities will ensure the accuracy of the approximants.

The critical exponent m_N can easily be found by using Newton's polygon algorithm. However, it is well known that, in the case of algebraic equations, the only singularities that are structurally stable are simple turning points. Hence, in practice, one almost invariably obtains $m_N = 1/2$. If we assume a singularity of algebraic type as in equation (11), then the exponent may be approximated by:

$$m_N = 1 - \frac{A_{2N}(\lambda_{CN})}{DA_{3N}(\lambda_{CN})} \tag{18}$$

For details on the above procedure, interested readers can see Vainberg and Trenogin (1974), Makinde (2005), Makinde and Mhone (2006), etc. It is very important to note at this junction that the solution series for the flow characteristics mentioned in Section 3 can be easily generated in powers of either α or R. Hence, with respect to the problem under investigation, λ can either be α or R as shown in the following section.

5. Results and discussion

The bifurcation procedure above is applied on the first 24 terms of the solution series obtained in Section 3 and we obtained the results as shown in Tables I-III.

Table I shows the rapid convergence of the dominant singularity α_c, i.e. the divergent channel semi-angle at bifurcating point in the flow field for $R = 20.0$, $Rm = 0$ and $H = 1.0$ together with its corresponding critical exponent m_c with gradual increase in the number of series coefficients utilized in the approximants. Figure 2 shows the fluid radial velocity profile for divergent channels at varying semi-angles.

The presence of flow reversal near the channel's wall is noticed for divergent channel with very large semi-angle. As the divergent semi-angle decreases, a general decrease in the fluid velocity around the centreline region of the channel is observed and the flow reversal near the walls is suppressed. Figure 3 shows the effect of magnetic field on the fluid radial velocity profile for a divergent channel at semi-angle $\alpha = 45°$. It is interesting to note that an increase in the magnetic field intensity causes a general

d	N	α_c	m_c
1	9	0.26791645	0.4798161
2	12	0.26920722	0.4978656
3	15	0.26915666	0.4998759
4	18	0.26916217	0.4999999
5	21	0.26916246	0.5000000
6	24	0.26916246	0.5000000

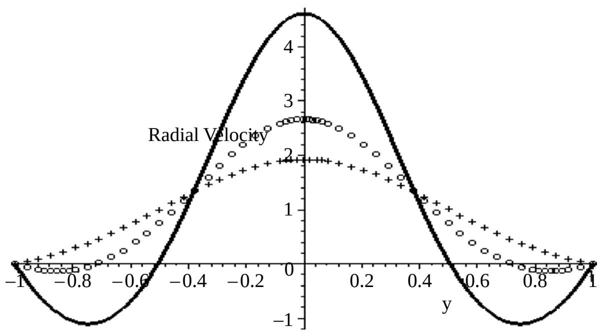
Table I. Computations showing the procedure rapid convergence for $H = 1.0$, $R = 20.0$, $Rm = 0$

H	0	1	3
$R_c(Rm = 0)$	54.4389	54.4717	54.6651
$R_c(Rm = 1)$	54.4389	54.4955	54.7875
$R_c(Rm = 2)$	54.4389	54.4966	54.8029
m_c	0.50000	0.50000	0.50000

Table II. Computation showing the divergent channel flow critical Reynolds number for bifurcation at $\alpha = 0.1$

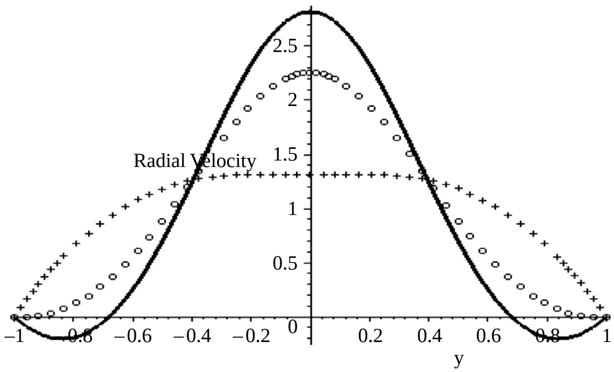
H	0	1	2	3	4	5
$\alpha_c(Rm = 0)$	0.267960	0.269162	0.272906	0.279878	0.290431	0.307406
$\alpha_c(Rm = 1)$	0.267960	0.269269	0.273361	0.280866	0.293082	0.313469
$\alpha_c(Rm = 2)$	0.267960	0.269375	0.273818	0.282075	0.295950	0.320828
m_c	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000

Table III. Computation showing the divergent channel critical semi-angles for bifurcation at $R = 20.0$



Notes: ___________ $\alpha = \pi/3$; oooooooo $\alpha = \pi/4$; +++++++ $\alpha = \pi/6$

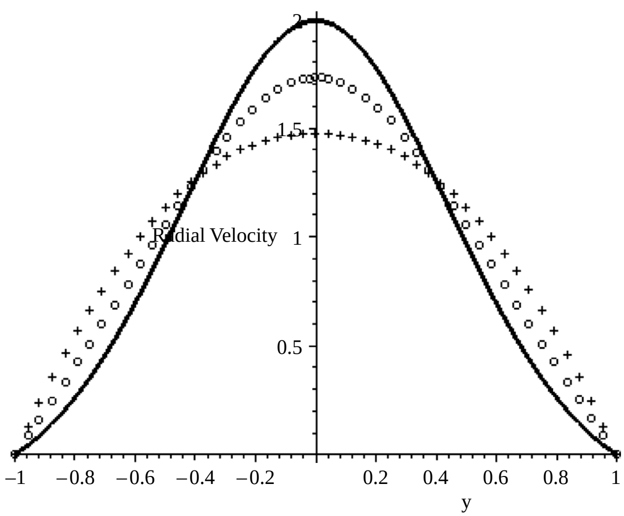
Figure 2. Fluid radial velocity profile for a divergent channel at varying semi-angles; $Rm = 1.0$; $H = 1.0$; $R = 7.0$



Notes: __________ H = 0; oooooooo H = 2.0; +++++++ H = 4.0

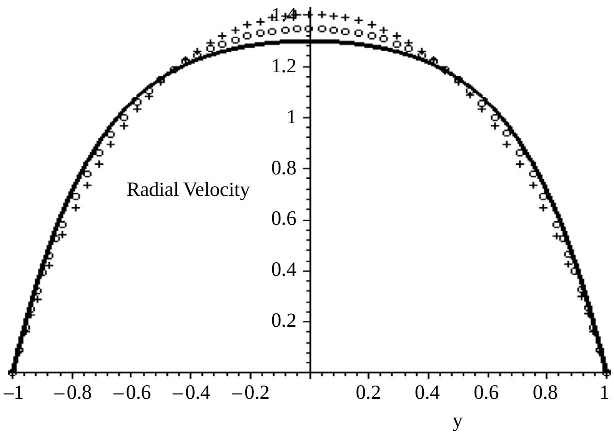
Figure 3.
Fluid radial velocity profile for a divergent channel at semi-angle $\alpha = \pi/4$; $Rm = 1$; $R = 7.0$

decrease in the fluid velocity around the centreline region of the channel. We also noticed that the Jeffery-Hamel flow corresponding to the case of $H = 0$ (no magnetic field) and shows flow reversal near both walls, i.e. internal boundary layer separation. As the Hartmann number (H) increases the flow reversal disappear, for $H = 2$, it has been suppressed already. The profile becomes distinctively convex for $H = 4$. We see that moderate values of H are sufficient to avoid the flow reversal that is typical of ordinary fluids in divergent channels. Figure 4 shows the effect of magnetic Reynolds number on the fluid radial velocity profile. It is noteworthy that an increase in the values of magnetic Reynolds causes a general decrease in the fluid velocity around the centreline region of the channel. The case of convergent channel (i.e. α = -45°) is shown in Figure 5. No reversal in the flow field near the walls occur in convergent channel, however, an increase in the magnetic Reynolds number causes a further increase in the fluid velocity around the channel centreline region. In Figure 6, we show a slice of the bifurcation diagram at $\alpha = 0.1$. The diagram suggests that, for this value of α, there is



Notes: __________ Rm = 0; oooooooo Rm = 1.0; +++++++ Rm = 2.0

Figure 4.
Fluid radial velocity profile for a divergent channel at semi-angle $\alpha = \pi/4$; $H = 3$; $R = 7.0$



Notes: __________ Rm = 0; oooooooo Rm = 1.0; +++++++ Rm = 2.0

Figure 5. Fluid radial velocity profile for a convergent channel at semi-angle $\alpha = -\pi/4$; $H = 3$; $R = 7.0$

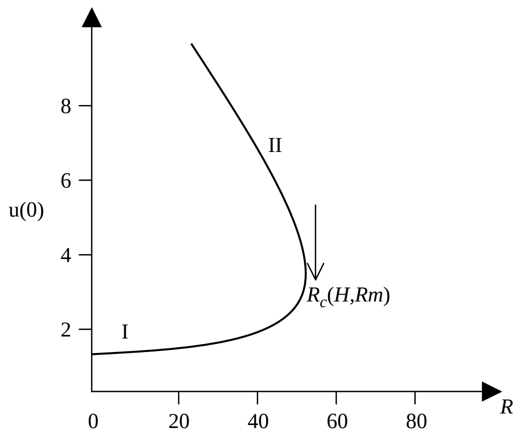


Figure 6. A slice of approximate bifurcation diagram in the (R, $u(0; \alpha = 0.1, R, Rm, H)$) plane

a turning point at $R_c(H, Rm)$. It is interesting to note that the magnitude of R_c increases with an increase in the values both magnetic field intensity and magnetic Reynolds number (i.e. H, Rm) as shown in Table II. For the case of classical Jeffery-Hamel flow (i.e. $H = 0$), we obtained $R_c(H = 0) \approx 54.4389$. This is in good agreement with Fraenkel's result, namely $R_c(H = 0) \approx 5.461/\alpha$ as $\alpha \rightarrow 0$. We remark that, as $\alpha \rightarrow 0$, the flow tends to plane Poiseuille flow. Consequently, as the flow tends to hydromagnetic plane Poiseuille flow, we observe that turning point in the flow field varies depending on the magnitude of magnetic field intensity (Table II). Another slice of the bifurcation diagram at $R = 20$ is shown in Figure 7. The diagram suggests that, for negative values of α (i.e. convergent channel), the solution is unique. We noticed that the solution bifurcates at $\alpha_c(H, Rm)$ as shown in Table III. The magnitude of α_c increases with an increase in the values both magnetic field intensity and magnetic Reynolds number. For the case of classical Jeffery-Hamel flow (i.e. $H = 0$),

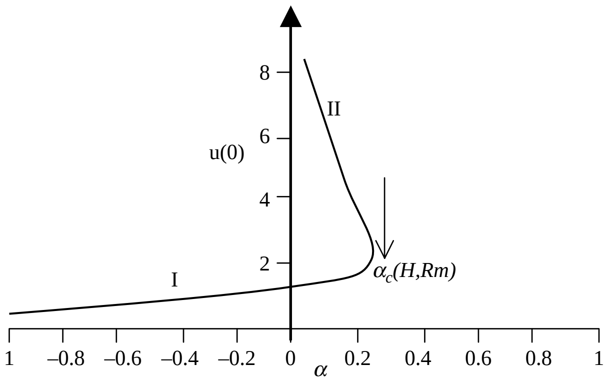


Figure 7.
A slice of approximate bifurcation diagram in the (α, $u(0; \alpha, R = 20, Rm, H)$) plane

we obtained $\alpha_c(H = 0) \approx 0.26796$. This is in good agreement with Khan *et al.* (2003) result, namely $\alpha_c(R = 20) \approx 0.27$. However, we remark here that this particular result of Khan *et al.* (2003) contains round up error. Finally, these bifurcation diagrams in Figures 6 and 7 show how the flow changes and bifurcates as the angle of inclination, the magnetic field intensity and the magnetic Reynolds number and the Reynolds number vary. In particular, for every α, there is a critical value $R_c(H, Rm)$ such that, for $0 \leq R(H, Rm) < R_c(H, Rm)$ there are two solutions (labelled I and II) and the solution II diverges to infinity as $R \rightarrow 0$.

6. Conclusion

The influence of arbitrary magnetic Reynolds number on 2D, steady flow of an incompressible conducting viscous liquid in convergent-divergent channels is studied using a new perturbation series summation and improvement technique. A bifurcation study by analytic continuation of a power series in the bifurcation parameter for a particular solution branch is performed. The procedure reveals accurately the analytical structure of the solution function and pertinent results for velocity field, flow reversal control and bifurcations are discussed quantitatively.

References

Baker, G.A. and Graves-Morris, P. (1996), *Padé Approximants*, Cambridge University Press, New York, NY.

Banks, W.H.H., Drazin, P.G. and Zaturska, M.B. (1988), "On perturbation of Jeffery-Hamel flow", *J. Fluid Mech.*, Vol. 186, pp. 559-81.

Batchelor, G.K. (1967), *An Introduction to Fluid Dynamics*, Cambridge University Press, Cambridge.

Fraenkel, L.E. (1962), "Laminar flow in symmetrical channels with slightly curved walls. I: on the Jeffery-Hamel solutions for flow between plane walls", *Proc. R. Soc. Lond. A*, Vol. 267, pp. 119-38.

Guttamann, A.J. (1989), "Asymptotic analysis of power – series expansions", in Domb, C. and Lebowitz, J.K. (Eds), *Phase Transitions and Critical Phenomena*, Academic Press, New York, NY, pp. 1-234.

Hamadiche, M., Scott, J. and Jeandel, D. (1994), "Temporal stability of Jeffery-Hamel flow", *J. Fluid Mech.*, Vol. 268, pp. 71-88.

Hamel, G. (1916), "Spiralförmige Bewgungen Zäher Flüssigkeiten", *Jahresbericht der Deutschen Math. Vereinigung*, Vol. 25, pp. 34-60.

Hunter, D.L. and Baker, G.A. (1979), "Methods of series analysis III: integral approximant methods", *Phys. Rev. B*, Vol. 19, pp. 3808-21.

Jeffery, G.B. (1915), "The two-dimensional steady motion of a viscous fluid", *Phil. Mag.*, Vol. 6, pp. 455-65.

Khan, M.A.H., Drazin, P.G. and Tourigny, Y. (2003), "The summation of series in several variable and its applications in fluid dynamics", *Fluid Dynamics Research*, Vol. 33, pp. 191-205.

Makinde, O.D. (1997), "Steady flow in a linearly diverging asymmetrical channel", *CAMES*, Vol. 4, pp. 157-65.

Makinde, O.D. (1999), "Extending the utility of perturbation series in problems of laminar flow in a porous pipe and a diverging channel", *J. Austral. Math. Soc. Ser. B*, Vol. 41, pp. 118-28.

Makinde, O.D. (2005), "Strongly exothermic explosions in a cylindrical pipe: a case study of series summation technique", *Mechanics Research Communications*, Vol. 32, pp. 191-5.

Makinde, O.D. and Mhone, P.Y. (2006), "Hermite-Padé approximation approach to MHD Jeffery-Hamel flows", *Applied Mathematics and computation*, Vol. 181, pp. 966-72.

Makinde, O.D. and Mhone, P.Y. (2007), "Temporal stability of small disturbances in MHD Jeffery-Hamel flows", *Computers and Mathematics with Applications*, Vol. 53, pp. 128-36.

Moreau, R. (1990), *Magnetohydrodynamics*, Kluwer Academic Publishers, Dordrecht.

Sergeyev, A.V. and Goodson, D.Z. (1998), "Summation of asymptotic expansions of multiple-valued functions using algebraic approximants: application to anharmonic oscillators", *J. Phys. A*, Vol. 31, pp. 4301-17.

Sobey, I.J. and Drazin, P.G. (1986), "Bifurcations of two-dimensional channel flows", *J. Fluid Mech.*, Vol. 171, pp. 263-87.

Tourigny, T. and Drazin, P. (2000), "The asymptotic behaviour of algebraic approximants", *Proc. Roy. Soc. London A*, Vol. 456, pp. 1117-37.

Vainberg, M.M. and Trenogin, V.A. (1974), *Theory of Branching of Solutions of Non-linear Equations*, Noordoff, Leyden.

Corresponding author
O.D. Makinde can be contacted at: makindeo@ul.ac.za